A New Technique for the Quasi-TEM Analysis of Conductor-Backed Coplanar Waveguide Structures

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***Abstract*—Numerically efficient and accurate formulae based on the spectral domain method for the analysis of conductor-backed coplanar waveguide structures are presented. Quasi-TEM parameters are obtained for these waveguide structures by using piecewise linear functions to approximate the potential distribution at the air-dielectric interface. Techniques such as nonuniform discretization and bound estimation are described which demonstrate shorter computational times. Results on the characteristic impedance calculation of standard coplanar waveguide are given to demonstrate the numerical accuracy and efficiency of the method presented here.**

I. Introduction

Due to the increasing popularity of coplanar waveguides [1] for the design of hybrid and monolithic microwave integrated circuits, the need for accurate characterization of the structures has increased. The solutions should take into account the effects of environmental constraints, such as shielding, conductor backing, line-to-line coupling, and coupled lines. Conductor backing is often introduced to improve both the mechanical strength (GaAs substrate is typically thin and fragile) and heat dissipation of the guide. Among these advantages, conductor backed coplanar waveguide (CBCPW) also allows easy implementation of mixed coplanar/microstrip circuits. There are also certain potential problems such as leakage of power into surface waves, that are introduced by the use of conductor backing [2]. Many papers have been devoted to the analysis of CBCPW based on the quasi-TEM approximation. Although it has been pointed out that the quasi-TEM assumption is valid only at zero frequency, the dispersion characteristics presented in [3] suggests that 1-percent accuracy in the effective dielectric constant can be maintained with this assumption up to 20 GHz for dimensionally small structures. Several quasi-static approaches have been reported, including conformal mapping [4], finite difference [5] and spectral domain methods [6]–[9]. In this paper, a numerically efficient and accurate technique for evaluating the quasi-TEM parameters of CBCPW structures is presented, which is based on the spectral domain method. This technique uses the same theory as presented by Sawicki and Sachse [8], the difference being in the choice of expansion and testing functions. The potential distribution across the conductor surfaces of the waveguide is modeled by a set of piecewise linear functions, that allows closed-form analytical expressions to be derived. The proposed method is equivalent to a variational method which gives the upper bound on capacitance and hence a lower bound on the characteristic impedance. Techniques to improve the computational efficiency of these formulae including the nonuniform discretization and bound estimation schemes are described.

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II. Method of Analysis

The cross-sectional view of the conductor-backed CPW configuration under analysis is shown in Fig. 1. If the quasi-TEM approximation is employed, our problem reduces to solving Laplace's equation in the space plane subject to the appropriate boundary conditions. In the spectral domain, Laplace's equation can be written as [10}

$$G^{-1}(\alpha)\phi(\alpha) = \rho(\alpha) \tag{1a}$$

$$G^{-1}(\alpha) = \varepsilon_0(\varepsilon_1+\varepsilon_2)\alpha \cdot \left\{1 + k\sum_{n=1}^{\infty} e^{-2nt\alpha} + \frac{2\varepsilon_2}{\varepsilon_1+\varepsilon_2}\sum_{n=1}^{\infty} e^{-2nh\alpha}\right\} \tag{1b}$$

where

$$\begin{aligned} k &= 0 && \text{(Open CBCPW)} \\ k &= \tfrac{2\varepsilon_1}{\varepsilon_1+\varepsilon_2} && \text{(Shielded CBCPW)} \end{aligned} \tag{1c}$$

$\phi(\alpha)$, $\rho(\alpha)$ and $G(\alpha)$ are the transform of the potential distribution, charge density distribution, and the static Green's functions in the Fourier domain, respectively. These quantities are evaluated at the air-dielectric interface, where the ground planes are assumed to be infinitely wide. All metallic strips are assumed to be infinitely thin and perfectly conducting.

In the spectral domain methods reported [6]–[9], the unknown potential distribution is usually expressed in terms of appropriate basis functions which incorporate the edge effect. High numerical accuracy has been demonstrated employing Bessel functions and Chebyshev polynomials as the basis functions. The integrals in the resulting equations thus obtained are usually evaluated by numerical integration leading to long CPU time. It is therefore very useful if a closed-form expression can be found for these integrals to gain better numerical accuracy and efficiency. Consider the single coplanar waveguide structure shown in Fig. 1, a possible solution is to

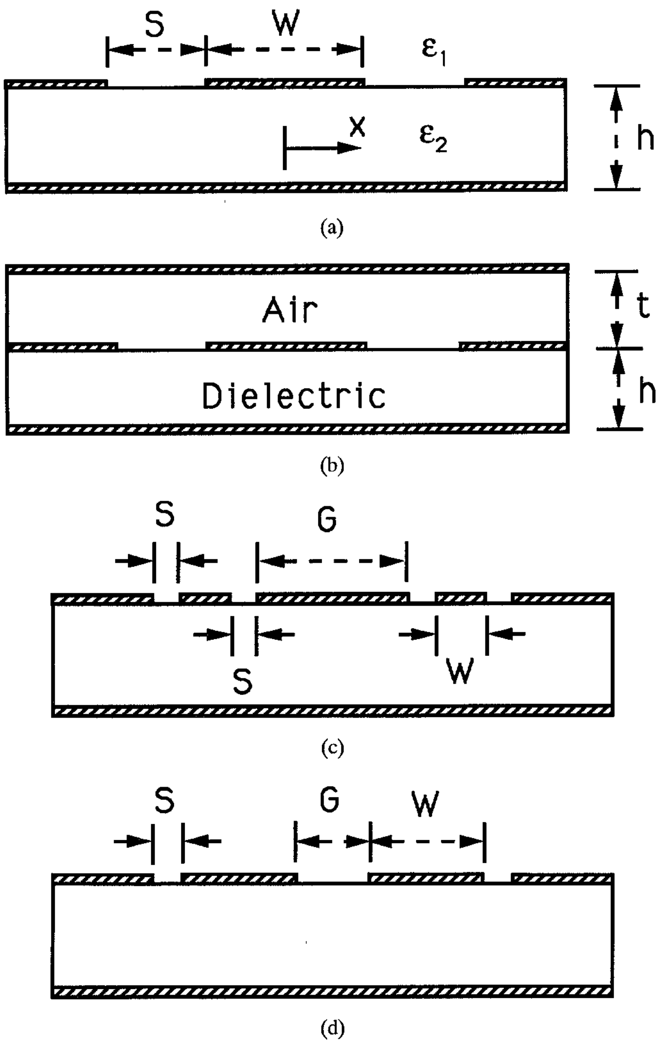

Fig. 1. Cross-sectional view of (1) open CBCPW, (b) shielded CBCPW, (c) Parallel lines in CBCPW, and (d) Coupled lines in CBCPW.

approximate the potential distribution $\phi(x)$ at the air-dielectric interface by a set of piecewise linear functions (Fig. 2):

$$\phi(x) = \sum_{i=1}^{N} K_i \phi_{a_i,a_{i-1}}(x) \tag{2a}$$

$$a_i = \frac{W}{2} + \zeta_i S$$
$$0 = \zeta_0 < \zeta_1 < \cdots < \zeta_{N-1} < \zeta_N = 1 \tag{2b}$$

$$\begin{aligned}\phi_{a,b}(x) &= 1 \quad (0 < x < b)\\ &= \frac{a-x}{a-b} \quad (b < x < a)\\ &= 0 \quad (a < x)\end{aligned} \tag{2c}$$

where K_i are the unknown coefficients to be determined, and ζ_i are predefined constants depending on the method of discretization. If the slot is divided uniformly, the value of ζ_i is equal to $1/N$. The number of sections needed could become rather large if a highly accurate solution is desired. Fortunately, by adopting a non uniform discretization scheme, the numerical accuracy of the solution can be improved without the need to increase the number of basis functions. An expression for ζ_i which has been found very useful for this purpose is shown below,

$$\zeta_i = 1 - \cos\left(\frac{i}{2N}\pi\right). \tag{3}$$

The principle behind this formula is that smaller sections are used over the region where the potential function is changing rapidly. The values of K_i can now be obtained by applying Galerkin's procedure [8] to (1), and together with the expression for the Fourier transform of (2c), given by,

$$\phi_{a,b}(\alpha) = 2\frac{\cos(b\alpha) - \cos(a\alpha)}{(a-b)\alpha^2}. \tag{4}$$

After some mathematical manipulations, the following equation is obtained:

$$\underline{A}\,\underline{K} = \frac{\pi}{\varepsilon_0(\varepsilon_1 + \varepsilon_2)}\underline{B} \tag{5a}$$

where $\underline{K} = [K_1 K_2 \cdots K_N]^T$ and $\underline{B} = [11 \cdots 1]^T$. The total electric charge per unit length of the center conductor has been assumed equal to 1 C/m. The element of matrix $\underline{A}$, denoted by $A_{i,j}$ $(i, j = 1, 2, \cdots, N)$, is then defined by,

$$\begin{aligned}A_{i,j} = &\int_0^\infty \alpha \phi_{a_i,a_{i-1}}(\alpha)\phi_{a_j,a_{j-1}}(\alpha)\,d\alpha\\ &+ k\sum_{n=1}^{\infty}\int_0^\infty \alpha e^{-2nt\alpha}\phi_{a_i,a_{i-1}}(\alpha)\\ &\cdot \phi_{a_j,a_{j-1}}(\alpha)\,d\alpha\\ &+ \frac{2\varepsilon_2}{\varepsilon_1+\varepsilon_2}\sum_{n=1}^{\infty}\int_0^\infty \alpha e^{-2nh\alpha}\phi_{a_i,a_{i-1}}(\alpha)\\ &\cdot \phi_{a_j,a_{j-1}}(\alpha)\,d\alpha.\end{aligned} \tag{5b}$$

Closed-form analytical solutions for the above expression have been derived and the resulting equation is shown below,

$$\begin{aligned}A_{i,j} = &F_0(a_i, a_j, a_{i-1}, a_{j-1})\\ &+ k\sum_{n=1}^{\infty} F_n\left(\frac{a_i}{2t}, \frac{a_j}{2t}, \frac{a_{i-1}}{2t}, \frac{a_{j-1}}{2t}\right)\\ &+ \frac{2\varepsilon_2}{\varepsilon_1+\varepsilon_2}\\ &\cdot \sum_{n=1}^{\infty} F_n\left(\frac{a_i}{2h}, \frac{a_j}{2h}, \frac{a_{i-1}}{2h}, \frac{a_{j-1}}{2h}\right)\end{aligned} \tag{6a}$$

$$F_n(a,b,c,d) = \frac{\begin{array}{c}I_n(c+b)+I_n(a+d)-I_n(a+b)-I_n(c+d)+\\ I_n(c-b)+I_n(a-d)-I_n(a-b)-I_n(c-d)\end{array}}{(a-c)(b-d)} \tag{6b}$$

$$I_n(p) = \frac{n^2}{2}\left\{(1-\beta^2)\ln(1+\beta^2) - 4\beta\arctan(\beta)\right\} \tag{6c}$$

where

$$\beta = \frac{p}{n}.$$

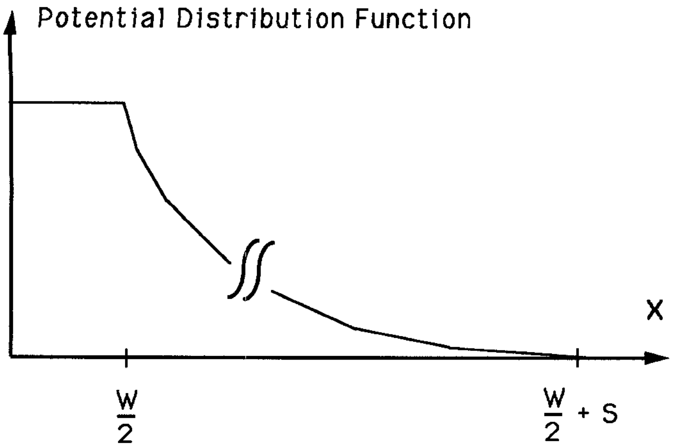

Fig. 2. Piecewise linear approximation of potential distribution at the air-dielectric interface of a single CBCPW transmission line.

The proper choice of basis function clearly eliminates the need for tedious numerical integration to compute the elements of $\underline{A}$. In order to keep the computational time and accuracy within reasonable bounds, it is necessary to accelerate the convergence of the infinite summations in (6a). It can be seen that the value of the sums are bounded, in the following relation:

$$\sum_{n=1}^{M-1} F_n(a,b,c,d) + \int_M^{\infty} F_n(a,b,c,d)\, da \le \sum_{n=1}^{\infty} F_n(a,b,c,d) \le \sum_{n=1}^{M-1} F_n(a,b,c,d) + \int_M^{\infty} F_n(a,b,c,d)\, dn \quad (7)$$

where the integrals in the upper and lower limits are evaluated as

$$\int_M^{\infty} F_n(a,b,c,d)\, dn = \frac{\begin{matrix} J(c+b)+J(a+d)-J(a+b)-J(c+d) \\ J(c-b)+J(a-d)-J(a-b)-J(c-d) \end{matrix}}{(a-c)(b-d)} \quad (8a)$$

$$J(p) = \frac{M^3}{6}\left\{ (3\omega^2 - 1)\ln(1+\omega^2) + 6\omega \arctan(\omega) + 2\omega^3 \arctan\left(\frac{1}{\omega}\right) - \pi\omega^3 \right\} \quad (8b)$$

where

$$\omega = \frac{p}{M}.$$

Note that the upper and lower bounds may be made to approach the "true" value of the infinite summation simply by increasing M. A formula which has been found very useful in approximating this "true" value is given below,

$$\sum_{n=1}^{\infty} F_n(a,b,c,d) \approx \sum_{n=1}^{M-1} F_n(a,b,c,d) + \frac{F_M(a,b,c,d)}{2} + \int_M^{\infty} F_n(a,b,c,d)\, dn. \quad (9)$$

Once the unknown coefficients are obtained, the characteristic capacitance C and impedance Z_o of the waveguide can then be determined from the expressions:

$$C = \left(\sum_{i=1}^{N} K_i\right)^{-1} \quad (10a)$$

$$Z_o = Z_a \sqrt{\frac{C_a}{C}}. \quad (10b)$$

Note that C_a and Z_a are the characteristic capacitance and impedance, respectively, of the same guide structure when all dielectric materials are replaced by a vacuum. These parameters may be calculated by conformal mapping theory. Although the above analysis is performed on a single coplanar waveguide structure, the same procedure can easily be extended to other configurations such as coupled lines, by employing an appropriate set of piecewise linear functions to model the potential distribution at the air-dielectric interface.

III. Numerical Results and Discussions

A computer program based on the formulae presented in this paper, has been developed to analyze the open CBCPW configuration shown in Fig. 1. In our first example, numerical results are generated to examine the accuracy of the solution using different discretization methods. Fig. 3 shows the plot of the characteristic impedance values as a function of the ratio S/h ($W/h = 1$, $\varepsilon_2 = 10$, $\varepsilon_1 = 1$). Based on the uniform discretization principle, curves are produced for $N = 1$, 3, 10 and 20. For the purposes of comparison, values obtained by the conformal mapping theory and the non uniform discretization method ($N = 4$) are also included in the diagram. It should be noted that the mapping theory produces quite large errors as the slot width is increased. Upon examining curves $U - 10$, $U - 20$, and $N - 4$, it is quite easy to see that the number of basis functions needed is reduced by a factor of between 4 and 5, using the nonuniform discretization formula given in (3). This is in fact a significant savings in terms of computational time and storage requirement since both of these parameters are roughly proportional to N^2.

In Fig. 4, the convergence rates for the evaluation of characteristic impedance versus the number of terms taken by the direct sum and bound estimation methods are shown. In each case the "relative error" is calculated by comparing the result to a series evaluation which has been computed to machine precision. Numerical experiments reveal that the method of direct sum requires over 200 terms to achieve 0.1% inaccuracy. However, this number is reduced to 3 for the same precision, by using the formula in (9). This indicates an improvement factor of almost a 100 in numerical efficiency.

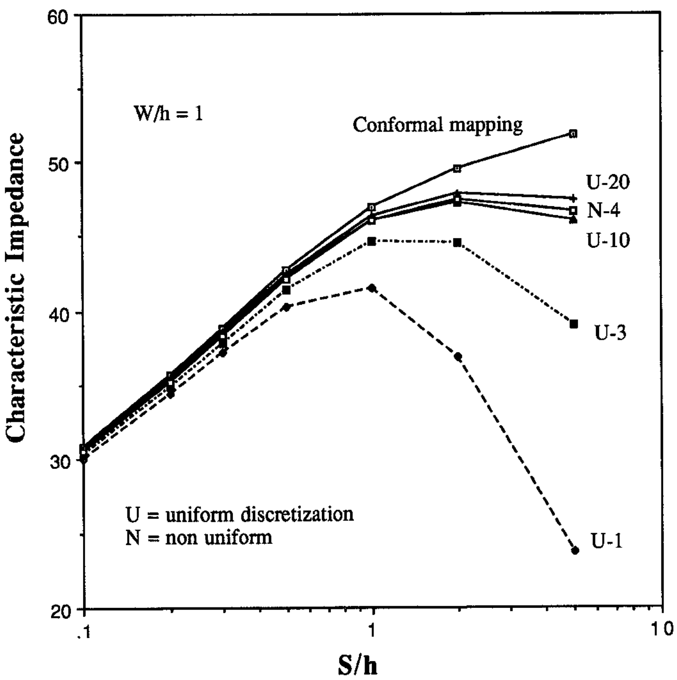

Fig. 3. Plot of characteristic impedance values versus the ratio S/h for the open CBCPW structure shown in Fig. 1.

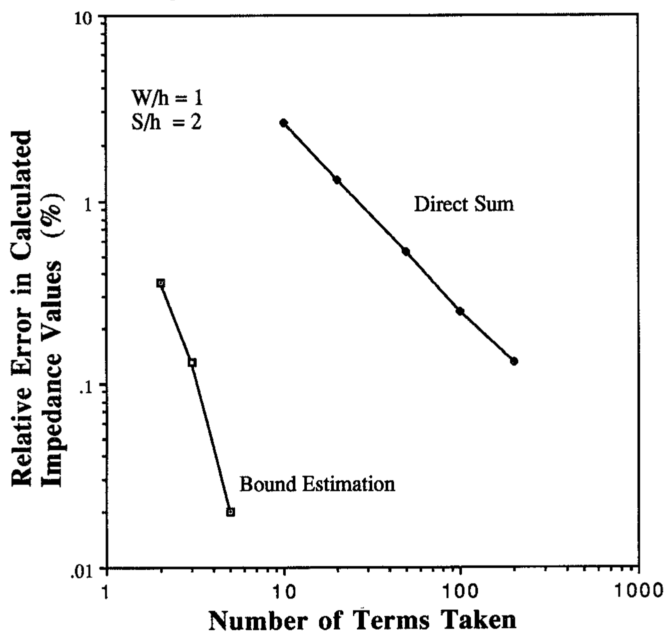

Fig. 4. Plot of relative error in characteristic impedance evaluation versus number of terms taken using direct and bound estimation methods for the open CBCPW structure shown in Fig. 1.

Finally, to illustrate the accuracy of this method, comparisons with respect to the ratio λ_g/λ of the open CBCPW configuration have been made and shown in Fig. 5. The last column of Fig. 5 shows the computed values (low frequency) reported by Shih and Itoh, which were obtained by a rigorous spectral domain approach. Upon examining the two set of data, it can be concluded that the accuracy of the formulas presented here is better than 0.5 of a percent. It should also be pointed out here that the numerical computations have been carried out on a Vax Cluster 8700 machine, and the computing time for each value is about 17 ms, with $N = 4$ and $M = 3$.

h (μm)	λg/λ this method	λg/λ Ref. [3]
50	0.319	0.319
100	0.339	0.339
150	0.352	0.351
300	0.368	0.368
500	0.374	0.373

W = 200 μm , S = 100 μm , εr = 13

Fig. 5. Comparisons of computed λ_g/λ values (low frequencies) with a rigorous spectral domain method for the open CBCPW structure shown in Fig. 1.

IV. Conclusion

A numerically efficient method for obtaining the quasi-TEM parameters of conductor-backed coplanar waveguide structures has been proposed. It can be seen that this approach is both easy to implement and rapidly convergent, thus making it an excellent choice for use in microwave CAD tools. The numerical results presented agree well with previously published data which were based on a full-wave analysis approach. The method of analysis described here may also be applied to the modeling of multiconductor systems.

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